# Some Topological Aspects of Spacetime Metrics

**A Synopsis**

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# 1 Introduction

The Euclidean space is an appropriate mathematical model to study the Classical Mechanics in which time is taken as universal and does not depend on the state of motion of observer but this theory found to be inconsistent with the propagation of light as electromagnetic radiation. In 1905, Albert Einstein proposed the theory of Special Relativity which overcomes this drawback. Hermann Minkowski, a German mathematician, one of Albert Einstein's teachers, observed that the Special Theory of Relativity could be understood geometrically as a theory of the four dimensional spacetime (later known as Minkowski space). But this theory is applicable only when gravity of the observer is neglected. Thus Minkowski spacetime is generally regarded as the appropriate arena to formulate those laws of the physics that do not refer specifically to gravitational phenomena and is sometimes mentioned as a flat spacetime. To get a better understanding of the physical universe, Albert Einstein established theory of General Relativity which describes gravitation in terms of the curved four dimensional spacetime. The four dimensional Lorentz manifold is one of the most important models for this theory [33].

In 1915, Kerl Schwarzschild solved the Einstein field equations which describe the geometry of spacetime due to the influence of mass and from there derived the Schwarzschild metric. This metric relies on curvature of spacetime to provide means of measuring invariant spacetime intervals around an isolated, static and spherically symmetric mass M, which could represent a star or a blackhole [25].

String Theory is consistently defined in ten dimensions, six of which are curled up in some small internal compact manifold. The procedure of linking this manifold to four dimensional physics is called string compactification. The idea of compactification arises naturally to resolve the discrepancy between the critical ten dimensions and the number of observed four dimensions. Since $M_{10}$ is dynamical, there can be solutions, consistent with the requirements imposed by local scale invariance on the world-sheet, which make the world appear four-dimensional. The simplest possibility is to have a background metric such that spacetime takes the product form $M_{10} = M^4 \times K_6$, where $M^4$ is four dimensional Minkowski space and $K_6$ is a compact space which is called internal manifold [20].

The simplicity and elegancy of Minkowski space provides a unified and logically consistent picture of numerous physical phenomena in the domain of relativity, Quantum Field Theory and String Theory. Moreover, to have escaped detection, $K_6$ must have dimensions of size smaller than the length scales already probed by particle accelerators. The type of theory observed in $M^4$ will depend on properties of the compact

space . For instance, in the classic analysis of superstring compactification, it was found that when $K_6$ is a Calabi-Yau manifold, the resulting four-dimensional theory has a minimal number of supersymmetries [20].

In 1919, the German mathematician Theodor Kaluza developed a theory of five-dimensional spacetime, which has a consequence of electromagnetism in the unseen fifth dimension. Consequently, in 1926 the Swedish mathematician Oskar Klein reconsidered Kaluza's theory and made some important improvements that also seemed to have applications in the emerging area of quantum theory. Since that time, theories involving extra hidden (or compactified) dimensions have become known as Kaluza-Klein theories.

The aim of the proposed work is to introduce and study topologies on spacetime using metrics of general relativity and Kaluza-Klein metric.

## 2 Notation and Preliminaries

This section comprises notation, basic definitions and known results which are used later. For terms which are not explained here, we refer to [13, 15, 26, 27, 28, 34, 35, 37]. The symbols $R$ and $N$ denote the set of real and natural numbers respectively.

### 2.1 Metrics

The metric tensor is an important object in curved space. It contains all the information required to describe the curvature of manifold. It gives the notion of "past" and "future" and determines the shortest distance between two points etc. It is denoted by $g_{\mu\nu}$, where $g_{\mu\nu}$ is usually non-degenerate, that is $g = |g_{\mu\nu}|$ does not vanish. It gives that the inverse metric $g^{\mu\nu}$ can be defined by $g^{\mu\nu}g_{\nu\sigma} = g_{\lambda\sigma}g^{\lambda\mu} = \delta^{\mu}_{\sigma}$.

#### 2.1.1 Euclidean metric

Euclidean line element in a three dimensional space with cartesian coordinates is given by $ds^2 = dx^2 + dy^2 + dz^2$.

#### 2.1.2 Lorentzian metric

Lorentz inner product is a bilinear form $g : R^n \times R^n \longrightarrow R$ satisfying the following properties (i) symmetric, i.e. $g(x,y) = g(y,x)$ for all $x, y \in R^n$ (ii) non-degenerate, i.e. if $g(x,y) = 0$ for all $y \in R^n$ then $x = 0$ and (iii)

there exists a basis $\{e_0, e_1, ..., e_{n-1}\}$ of $R^n$ such that $g(e_i, e_j) \equiv \eta_{ij}$ =1, for $i = j = 0$; -1, for $i = j = 1, ..., n-1$ and 0 otherwise. The matrix $\eta_{ij}$ is known as the Lorentzian metric.

$R^n$ with the Lorentzian metric is called the *n -dimensional Minkowski space* and is denoted by $M$.

### 2.1.3 Schwarzschild metric

In General Relativity, the unique spherically symmetric vacuum solution is the **Schwarzschild metric**. It gives the geometry of the vaccum spacetime outside a spherical star. It is determined by the mass M and has the line element in spherical coordinates $\{t, r, \theta, \phi\}$, as follows: $ds^2 = -(1 - \frac{2GM}{r})dt^2 + (1 - \frac{2GM}{r})^{-1}dr^2 + r^2 d\Omega^2$, where $d\Omega^2$ is the metric on a unit two-sphere and is given by $d\Omega^2 = d\theta^2 + \sin^2\theta d\phi^2$ and $G$ is the gravitational constant. The term $R_s$ is defined as the schwarzschild radius and is given by $R_s = 2GM$. Note that, as $M \to 0$ we recover Minkowski space, also the metric become progressively Minkowskian as $r \to \infty$, this property is known as **asymptotic flatness** [28].

### 2.1.4 Kaluza-Klein metric

Let $G_{ab}$ be the metric for a (4+d)- dimensional spacetime with coordinates $X^a$, where indices $a, b$ run from 0 to d+3. The five-dimensional line element is given by $ds^2 = G_{ab}dX^a dX^b = g_{\mu\nu}(x)dx^\mu dx^\nu + b^2(x)\gamma_{ij}(y)dy^i dy^j$, where $x^\mu$ are coordinates in four-dimensional spacetime with metric $g_{\mu\nu}, y^i$ are coordinates on the extra-dimensional manifold, taken to be maximally symmetric space with metric $\gamma_{ij}$ and $b$ is the extra dimensional scale factor which is independent of $y^i$ [28].

If the extra-dimensional manifold is flat ($\kappa = 0$) , the potential vanishes and there will only be massless scalar field. If there is curvature ($\kappa \neq 0$), the potential has no minimum. For $\kappa > 0$ the field will roll to $-\infty$, while for $\kappa < 0$ the field will roll to $+\infty$. But $\phi \propto \lg b$ ,where $\phi$ is known as the dilaton or radion which characterizes the size of the extra dimensional manifold. So this means that the scale factor $b(x)$ of the extra dimensions either shrinks to zero or becomes arbitrarily large, in either case ruining the hope for stable extra dimensions. Stability can be achieved, however, by choosing an appropriate matter Lagrangian and an appropriate field configuration in the extra dimensions [28].

### 2.1.5 Robertson-Walker metric

The Robertson-Walker metric is given by - $ds^2 = -dt^2 + R^2(t)\gamma_{ij}(u)du^i du^j$, where $(u^1, u^2, u^3)$ are coordinates on maximally symmetric three manifold space with metric $\gamma_{ij}$ and $R$ is the scale factor which tell us how big

the spacelike slice at the moment of time $t$ [28].

## 2.2 Topologies induced by Lorentzian metric

A vector $x = (x^0, x^1, x^2, ..., x^{n-1}) \in M$ is called *time-like* or *light-like* or *space-like*, according to $((x^0)^2 - (x^1)^2 - ..... - (x^{n-1})^2) > 0 \; or \; = 0 \; or \; < 0$. The sets $C^T(x) = \{y \in M : y = x \; or \; y - x \; is \; a \; time - like \; vector\}$, $C^L(x) = \{y \in M : y - x \; is \; a \; light - like \; vector\}$ and $C^S(x) = \{y \in M : y = x \; or \; y - x \; is \; a \; space - like \; vector\}$ are likewise respectively called the *time cone*, *light cone* and *space cone* at $x \in M$. A straight is called a time-like, light-like or space-like according as it is parallel to time-like, light-like or space-like vector and a space-like hyperplane is a hyperplane such that each of its non-zero vector is space-like.

Some topologies on $M$ are described below: The *Euclidean topology* on Minkowski space $M$ is the topology generated by the basis $B = \{N_\varepsilon^E(x) : \varepsilon > 0, x \in M\}$ where $N_\varepsilon^E(x)$ denotes the Euclidean neighbourhood. The *fine topology* is defined as the finest topology that induces the Euclidean topology on every timelike like and space-like hyperplane [5], while the space topology is the finest topology that induces the Euclidean topology on every space-like hyperplane and the time topology is the finest topology that induces the Euclidean topology on every time-like line. The f-topology [6] , s-topology [31] and t-topology [31] are the topologies generated by the basis $\{N_\varepsilon^E(x) \cap (C^T(x) \cup C^S(x)) : \varepsilon > 0, x \in M\}$, $\{N_\varepsilon^E(x) \cap C^S(x) : \varepsilon > 0, x \in M\}$ , $\{N_\varepsilon^E(x) \cap C^T(x) : \varepsilon > 0, \; x \in M\}$ respectively where $N_\varepsilon^E$ is the Euclidean ball centered at $x$ with radius $\varepsilon > 0$. These topologies were introduced by Zeeman.

The *A -topology* is the finest topology on Minkowski space which induces Euclidean topology on every timelike and lightlike line and spacelike hyperplanes [32].

The Zeeman topology induces manifold topology on every timelike geodesic and spacelike hypersurface [21], path topology is defined as the finest topology which induces manifold topology on every timelike curve [34], space topology is defined as the finest topology which induces manifold topology on every spacelike curve [27] and separating topologies are some of the physically significant topologies on Lorentz manifolds defined using the Lorentz metric as generalizations of the fine topology, t-topology, s-topology and f -topology, respectively.

## 2.3 Important theorems on Differentialbility

The derivatives are fundamental tools of Calculus. There are different types of problems of derivatives like physical and geometrical. Some important theorems related to derivatives are given below [35] :

1. If $f$ is differentiable at $c$, then $f$ is continuous at $c$.

2. **Chain Rule:** Let $g : R^n \to R^p$ and $f : R^p \to R^m$. Assume that $g$ is differentiable at $a$, with total derivative $g'(a)$. Let $b = g(a)$ and assume that $f$ is differentiable at $b$, with total derivative $f'(b)$. Then the composite function $h = f \circ g$ is differentiable at $a$ and the total derivative $h'(a)$ is given by $h'(a) = f'(b) \circ g'(a)$,the composition of the linear functions $f'(b)$ and $g'(a)$.

3. **Mean Value Theorem :** Let $S$ be an open subset of $R^n$ and assume that $f : S \to R^m$ is differentiable at each point of $S$. Let $x$ and $y$ be two points in $S$ such that $L(x,y) \subseteq S$, where $L(x,y) = \{tx + (1-t)y : 0 \leq t \leq 1\}$ denotes the line segment joining two points $x$ and $y$ in $R^n$. Then for every vector $a$ in $R^m$ there is a point $z$ in $L(x,y)$ such that $a \cdot \{f(y) - f(x)\} = a \cdot \{f'(z)(y-x)\}$.

4. **Implicit Function Theorem:** Let $f = (f_1, ..., f_n)$ be a vector-valued function defined on an open set $S$ in $R^{n+k}$ with values in $R^n$. Suppose $f \in C'$ on $S$. Let $(x_0; t_0)$ be a point in $S$ for which $f(x_0; t_0) = 0$ and for which the $n \times n$ determinant $\det[D_j f_i(x_0; t_0)] \neq 0$. Then there exists a $k$-dimensional open set $T_0$ containing $t_0$ and a unique vector-valued function $g$, defined on $T_0$ and having values in $R^n$, such that

   (a) $g \in C'$ on $T_0$,

   (b) $g(t_0) = x_0$,

   (c) $f(g(t); t) = 0$ for every $t$ in $T_0$.

5. **Inverse Function Theorem:** Assume $f = (f_1, ..., f_n) \in C'$ on an open set $S$ in $R^n$ and let $T = f(S)$. If the jacobian determinant $J_f(a) \neq 0$ for some point $a$ in $S$, then there are two open sets $X \subseteq S$ and $Y \subseteq T$ and a uniquely determined function $g$ such that

   (a) $a \in X$ and $f(a) \in Y$,

   (b) $Y = f(X)$,

   (c) $f$ is one-to-one on $X$

   (d) $g$ is defined on $Y$, $g(Y) = X$ and $g[f(x)] = x$ for every $x$ in $X$,

   (e) $g \in C'$ on $Y$.

## 3 Survey of the Relevant Literature

In this section, the study related to the proposed work as found in literature, has been presented.

Physically relevant non-Euclidean topologies on Minkowski space were first defined by Zeeman in 1967, namely $t$-topology, $s$-topology, $f$-topology, space topology, time topology and fine topology. Fine topology is the finest topology that induces the 3-dimensional Euclidean topology on every spacelike hyperplane and 1-dimensional Euclidean topology on time axis. It enjoys properties like connectedness and locally connectedness. However, it is not locally compact. Its group of homeomorphisms is equal to the group generated by the Lorentz group together with dilatations and translations. Nanda proved this conjecture for space topology [30], $t$-topology [31] and $s$-topology [31]. He [30] also found that $M$ is not normal, not locally compact and not second countable in the context of both space topology and $s$ topology. Further, Agrawal et al. (2012) [8] obtained a characterization of the compact sets of $n$-dimensional Minkowski space with $s$-topology and proved it to be not simply connectedness for 2-dimensional case using the notion of Zeno sequences which was initially introduced by Zeeman for fine topology [5]. Also, $n$-dimensional Minkowski space with $s$-topology was obtained to be a separable, first countable, path-connected, not regular, not metrizable, non second countable, not compact and not Lindelöf space. Continuing the topological study of $n$-dimensional Minkowski space with the $t$-topology, denoted by $M^t$, Agrawal et al.[7] characterized its compact sets and proved it to be a Hausdorff, path connected, separable, first countable but not second countable, not normal, not compact, not locally compact, not paracompact, not metrizable and not locally $n$-Euclidean space. Afterwards, Agrawal et al. (2015) obtained that invariance of domain theorem does not hold in Minkowski space with the $t$-topology unlike the Euclidean case. But its consequence, i.e. no nonempty open subset of the $n$-dimensional Minkowski space with $t$-topology is homeomorphic to an open subset of the $m$-dimensional Minkowski space with $t$-topology for $n \neq m$. The technique to prove these results is entirely different from that of Euclidean case [10]. Quite recently, S. Erosy [29] studied the interior and closure of some important subsets of Minkowski space with respect to the $s$-topology and Euclidean topology. Continuing the topological study of Minkowski space, Agrawal et al. [6] studied the Minkowski space with the $f$-topology and obtained the characterization of its compact sets using the notion of Zeno sequances. It was obtained to be Hausdorff, path connected, separable, first countable, non-regular, non-second countable, non-paracompact, non-Lindelöf, non-normal, non-locally compact and non-metrizable space. For the two dimensional case, it was obtained to be a non-simply connected space.

Over the past few decades, there have been various attempts at defining the physically relevant topologies on spacetime manifolds. The most natural topology on manifolds is the manifold topology. But this topology is inappropriate as (i) it does not give the distinction between the spatial and temporal directions, (ii) its homeomorphism group is very vast and consist many undesirable maps (iii) the set of continuous path contains many unphysical paths that allows switching between temporal and spatial direction. This leads to define the non-manifold topologies on spacetime. Göbel, in 1976, defined general relativistic analogue of the fine topology (which he calls Zeeman topology) and obtained its group of homeomorphisms to be the same as the group of homothetic transformations on spacetime and found its world lines to be the broken geodesics [21]. In the same year, a seminar of Rüdiger Göbel at Cambridge stimulated Hawking, King and McCarthy to introduce a physically relevant non-manifold path topology on strongly causal spacetime. It is found to be more appealing and manageable topology than Zeeman topology. It has been found that it is first countable, separable, Hausdorff, path connected, locally path connected but not regular, not normal, not locally compact and not paracompact. Also, it determines the causal, differential and conformal structure of spacetime. Malament [2] proved that this result is true for every spacetime. Naber [13] found that path topology on four dimensional Minkowski space, a strongly causal spacetime, is same as the Zeeman´s $t$-topology. Low [24] proved that Minkowski space with $t$-topology is not simply connected and generalized this result to Lorentz manifolds with path topology.

Göbel [23] defined infinite variety of non-manifold topologies on Lorentz manifolds and remarked that all these topologies are Hausdorff, locally and globally path connected and none of these topologies is regular, normal, locally compact and paracompact without given proofs. One of these topology is recognized as generalization of time topology on Minkowski space. He obtained Zeeman's conjecture in the context of time topology on Minkowski space as a consequence of the result obtained for its generalize case on Lorentz manifolds. An alternate proof is obtained by Agrawal et al. in [9] using a simpler approach similar to one used in [13] for the computation of homeomorphism group of Minkowski space with path topology. Fullwood [4] worked out that the need of manifold topology to define continuous timelike curves (used to define path topology) restricts the class of spacetime on which the path topology can be defined. He suggested a new topology on spacetime by combining the idea of Hawking's path topology and Alexandrov topology, which is completely defined in terms of causal structure and also proved that distinguishing condition holds on a manifold if and only if this new topology is equivalent to path topology. Also found that homeomorphism group of this new topology on the spacetimes having chronology condition is the group of chronological

and antichronological bijections and proved that this new topology determines the causal, differential and conformal structure for the spacetimes having distinguishing condition.

Extra dimension metric has been introduced by Kaluza in 1919 which has reformulated by Klein in 1926.In 1966, Werner Israel [36] has presented new analytic extensions of the Schwarzschild and Reissner-Nordström manifolds in which the metric coefficients are simple rational functions of the coordinates and the complete manifold is covered by a single coordinate patch. A study of radial time-like geodesics leads naturally to an interpretation of the domain inside the Schwarzschild "singularity" involving a breakdown of the global distinction between past and future in this region.

In 2002, M. Arik et al. [16] invastigated a rotationally and translationally invariant metric in Finsler space. They had rewrite the metric in Riemanian space by increasing the dimension of spacetime and introduced additional coordinates such that for specific values of these coordinates, the geodesics of the four dimensional Finslerian spacetime and six dimensional Riemanian spacetime are identical. Cosmological solutions described by this metric give rise to an equation of state corresponding to a space dominated by domain walls and an internal space dominated by strings.

In 2003, Holger Gies [14] analyzed the possibility of nonperturbative renormalizability of gauge theories in $D > 4$ dimensions. He developed the conditions, that allows for renormalizability in extra dimensions owing to a non-Gausian ultraviolet stable fixed point. His conditions predicts a critical dimension $D_{cr}$ beyond which the UV fixed point vanishes, such that renormalizability is possible for $D \leq D_{cr}$. His results excluded the nonperturbative renormalizability of gauge theories in $D = 6$ and higher dimensions.

In 2008, Nima Arkani-Hamed et al. [19] showed that the theory of Origination of small couplings in a low-energy theory from symmetries in a high-energy theory is violated in theories where the Standard Model fields are confined to a thick wall in extra dimensions, with the fermions "stuck" at different points in the wall. Couplings between them are then suppressed due to the exponentially small overlaps of their wave functions. This provides a framework for understanding both the fermion mass hierarchy and proton stability without imposing symmetries, but rather in terms of higher dimensional geography. A model independent prediction of this scenario is non-universal couplings of the Standard Model fermions to the "Kaluza-Klein" excitations of the gauge fields. This allows a measurement of the fermion locations in the extra dimensions at the LHC or NLC if the wall thickness is close to the TeV scale.

In 2015, David Bate [3] proved the equivalence of two seemingly very different ways of generalising Rademacher's theorem to metric measure spaces. One such generalisation is based upon the notion of

forming partial derivatives along a very rich structure of Lipschitz curves in a way analogous to the differentiability theory of Euclidean spaces. This approach to differentiability in this generality appears here for the first time and by examining this structure further, he had arrived to several descriptions of Lipschitz differentiability spaces.

In 2015, Guy C. David [11] studied the Lipschitz differentiability spaces, a class of metric measure spaces introduced by Cheeger in [Ch99]. He showed that if an Ahlfors regular Lipschitz differentiability space has charts of maximal dimension, then, at almost every point, all its tangents are uniformly rectifiable. In particular, at almost every point, such a space admits a tangent that is isometric to a finite-dimensional Banach space. In contrast, He has also proved that if an Ahlfors regular Lipschitz differentiability space has charts of non-maximal dimension, then these charts are strongly unrectifiable in the sense of Ambrosio-Kirchheim.

In 2017, Murat Altunbas et al. [17] have been investigated the differential equations of geodesics; parallelism, incompressibility and closeness conditions of the horizontal and complete lift of the vector fields with respect to Kaluza-Klein metric on tangent bundle.

In 2018, Michael Kalisch [18] had worked for the black hole solutions in Kaluza-Klein theory. Considering only the static case without electric charge, it shows that there is a much richer phase space than in the usual four-dimensional theory, where only the Schwarzschild solution exists. There are at least two types of solutions with a completely different horizon topology first is : localized black holes with an ordinary spherical horizon and black strings with a horizon that wraps the compact dimension. This has been proved that the solution branches of both types are connected via a singular topology changing solution that is controlled by the double-cone metric. He has studied the singular transit solution in five and six spacetime dimensions with the help of a highly accurate numerical scheme.

# 4 Motivation

In the context of listing physically relevant non-Euclidean topologies on Minkowski spacetime, fine, $t$-, $s$-, space and time topologies are some of the such topologies. The homeomorphism groups with respect to these topologies have been obtained to be isomorphic to the group generated by the Lorentz group,translations and dilatations. These are now known to be non-regular, path connected and not simply connected. Different dimensional spaces are not homeomorphic. Their compact sets differ largely from those of Euclidean spaces. Hawking et al. defined path topology on Lorentz manifolds which was found to be the same as the $t$-topology. Most of the topologies on Minkowski spacetime have been generalized to Lorentz manifolds as their general relativistic analogues using exponential map [5, 6, 7, 8, 10, 12, 21, 22, 23, 24, 30, 33, 34].

There are many applications of topological study. To quote a few, homotopy groups are used to classify topological solitons in Quantum Field Theory [1]. David J. Thouless of University of Washington, F. Duncan M. Haldane of Princeton University and J. Michael Kosterlitz of Brown University were awarded the 2016 Nobel Prize in Physics for their discovery of topological phase transitions and topological phases of matter using topological invariants.

Einstein established the theory of space, time and gravity which is known as the General Theory of Relativity. In this theory, gravity plays a role of the curvature of spacetime which helps in the study of the geometry of spacetime. However, General Theory of Relativity does not provide the complete understanding of spacetime.

One way to modify the theory of General Relativity is to allow for the existence of extra spatial dimensions. Such theories are known as Kaluza-Klein theories. As the Minkowskian metric induces topologies, in the same way topologies are proposed to be defined using metrics of General Theory of Relativity and Kaluza-Klein theories.

The notion of differentiation of functions on Euclidean spaces, i.e. $R^n$ is well known. A natural question is to examine the effect of replacing the Euclidean topology by a non-Euclidean topology, on the notion of differentiation. This study is proposed to be carried out in the present proposal.

# 5 Proposed Direction of Work

Several topologies have been introduced using the Lorentz metric on Minkowski space, the spacetime of special theory of relativity and generalized to Lorentz manifolds. Topological properties, invariance of domain theorem and compact sets with regard to these are now well known as presented in the previous sections. General theory of relativity is Einstein's theory of spacetime and gravitation where gravity is a manifestation of the curvature of spacetime and the geometry of spacetime is encoded in its metric. Kaluza-Klein models of spacetime permits the existence of extra dimensions and give a more accurate picture of the physical universe.

In this light, it is proposed to introduce topologies on spacetimes using metrics of general relativity and Kaluza-Klein theories and to study some of the yet unexplored aspects of topologies defined using spacetime metrics. Specific objectives are as follows:

1. To introduce and study the notion of limit of functions from Minkowski spacetime to itself.
2. To introduce and study the notion of differentiability on Minkowski spacetime and obtain related results.
3. To introduce topologies on spacetimes using metrics of general relativity and Kaluza-Klein theories.
4. To study the respective homeomorphism groups.
5. To study various topological properties, namely connectedness, path connectedness, separability, first countablity, second countablity etc.
6. To study the relationship between newly introduced topologies and the existing topologies on Lorentz manifolds.

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